Trigonometry Solution

1) 1 rev = 400^g

$360° = 400^g$

$90° = 100^g$

B - option

2) $\frac{D}{90°} = \frac{G}{100^g}$

given grade, $G = 200^g$

Degree, $D = ?$

$D = \frac{200^g}{100^g} \times 90°$

$D = 180°$ (none)

3) $s = r\theta$ ~~(none)~~

4) or $l = r\theta$ (θ in rad)

c - option

4) $\theta = 30° = \pi/6$ rad

$r = 2$

l of arc $= s = r\theta = 2 \times \frac{\pi}{6}$

$s = \pi/3$

5) $1 \text{ rad} = \frac{180°}{\pi}$

$\pi/12 \text{ rad} = \frac{180°}{\cancel{\pi}} \times \frac{\cancel{\pi}}{12}$

$= 15°$ A-option

6) $1° = \frac{\pi}{180}$ rad

$30° = \frac{\pi}{180} \times 30$ rad

$30° = \frac{\pi}{6}$ rad

7) $10° 30' = 10° + \frac{30}{60}$

$= 10° + 0.5°$

$= 10.5°$

convert in radians

$= \frac{\pi}{180} \times 10.5$ (option A)

also

$= \frac{\pi}{180} \times \frac{21}{2} = \frac{\pi 21}{360}$

$= \frac{7\pi}{120}$ (option B)

so A and B both

8) $1° = \frac{\pi}{180}$ rad

and $1 \text{ rad} = \frac{180°}{\pi}$

$\frac{2\pi}{15} \text{ rad} = \frac{180°}{\cancel{\pi}} \times \frac{2\cancel{\pi}}{15}$

$= 24°$

9) $\sin(\alpha + \beta)$

$= \sin\alpha\cos\beta + \cos\alpha\sin\beta$

10) $1 \text{ rad} = 57° 17' 45''$

None

11) angle b/w hour hand and min hand at 4:30 pm

At 4:00pm hour-hand is at 4 which is $4 \times 30 = 120$ degree from 12.

In 30min it moves half distance b/w 4 and 5 so hr hand is at $120 + 15 = 135$ deedree from 12.

At 30min past the hr min hand is at $30 \times 6 = 180$ degree from 12

angle b/w both is $|180 - 135| = 45$ degree.

12) Sum of Internal angle of polygons is calculated by,

$(n-2) \times 180°$

$n \Rightarrow$ number of sides

$(6-2) \times 180°$

$= 720°$ Ans

13) For Internal angle

$= \frac{(n-2) \times 180}{n}$

$= \frac{(7-2) \times 180}{7} = \frac{900}{7} = 128.5°$

14) cosine function has a period of 2π (it repeat every 2π rad)

$\cos(2(0)\pi) = \cos 0 = 1$

$\cos(2(1)\pi) = \cos 2\pi = 1$

so $\cos(2n\pi) = 1$

15) $(2n+1)$ represents an odd number

$\cos(\pi) = -1 \quad (n=0)$

$\cos(3\pi) = -1 \quad (n=1)$

so $\cos(2n+1)\pi = -1$

16) $\sin(0) = 0 \quad (n=0)$

$\sin(2\pi) = 0 \quad (n=1)$

$\sin(2n\pi) = 0$ always

17) $\sin(\pi) = 0 \quad (n=0)$

$\sin(3\pi) = 0 \quad (n=1)$

so $\sin(2n+1)\pi = 0$

18) $\dfrac{\sin(-\theta)}{\cos(-\theta)} = \dfrac{-\sin\theta}{\cos\theta}$

$= -\tan(\theta)$ Ans

19) $\sin(270) + 5\cos(180) - 3\cos(270) = ?$

$= -1 + 5(-1) - 3(0)$

$= -1 - 5 = -6$ Ans

20) $\cos\theta = \dfrac{5 \rightarrow \text{Base}}{13 \rightarrow \text{hyp}}$

$P = \sqrt{H^2 - B^2} = \sqrt{13^2 - 5^2}$

$P = \sqrt{144} = 12$

θ in IVQ

$\tan\theta = \dfrac{P}{B} = \dfrac{12}{5} = -\dfrac{12}{5}$

21) skip -

22) $\sec\theta + \tan\theta = x$

$\sec\theta - \tan\theta = ?$

we have

$\sec^2\theta - \tan^2\theta = 1$

$(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

$x(\sec\theta - \tan\theta) = 1$

$\Rightarrow \sec\theta - \tan\theta = 1/x$

23) Allied angle Identity

$\cos(180^\circ - \theta) = -\cos(\theta)$

$\cos(180^\circ - 60^\circ) = -\cos 60^\circ$

24) $\dfrac{\sin(180-\theta)}{\cos(180-\theta)} = \dfrac{\sin\theta}{-\cos\theta}$

$= -\tan\theta$

25) True

26) θ and $180^\circ - \theta$

when added

$\theta + (180 - \theta) = 180^\circ$

Def of supplementary angles

27) skip -

28) $\cos(\alpha + \beta)$

$= \cos\alpha\cos\beta - \sin\alpha\sin\beta$

29) $\dfrac{\sin(13)\cos(77) + \cos(77)\sin(13)}{\cos(13)\cos(77) - \sin(13)\sin(77)}$

$= \dfrac{\sin(13+77)}{\cos(13+77)} = \dfrac{\sin(90)}{\cos(90)}$

$= \dfrac{1}{0}$ undefine or ∞

30) $\tan(2\pi - \theta) = -\tan\theta$

31) $\cos(2\pi - \theta) = \cos\theta$

32) $\sin(180 - \theta) = \sin\theta$

so $\sin(135)$

$= \sin(180 - 45) = \sin 45^\circ$

$\Rightarrow$ A option

33) $\sin(360 - \theta)$

$= -\sin\theta$

$\sin(300) = \sin(360 - 60)$

$= -\sin(60)$

$= -\dfrac{\sqrt{3}}{2}$ Ans

34) Find coterminal angle, $\cos(480^\circ) =$

$\cos(360 + 120) = \cos(120^\circ)$

$\cos(120^\circ) = \cos(180 - 60)$

$= -\cos 60^\circ = -\dfrac{1}{2}$

35) $\dfrac{3\pi}{8} + \dfrac{\pi}{8} = \dfrac{4\pi}{8} = \dfrac{\pi}{2}$

$\Rightarrow$ complementary

36) $\cos\left(\dfrac{\pi}{2} - \theta\right) = \sin\theta$

so $\sin\left(\dfrac{\pi}{8}\right) = \cos\left(\dfrac{\pi}{2} - \dfrac{\pi}{8}\right)$

$= \cos\left(\dfrac{3\pi}{8}\right)$ b-option.

37) $1050 - 360 = 690$

$690 - 360 = 330$

so $\sin(1050) = \sin(330)$

$\sin(330) = \sin(360-30)$

$= -\sin(30) = -\frac{1}{2}$

38) $\frac{9\pi}{4} - 2\pi = \frac{\pi}{4}$

so $\sin\left(\frac{9\pi}{4}\right) = \sin\frac{\pi}{4}$

$= \frac{1}{\sqrt{2}}$

39) $\cos(24) + \cos(55) + \cos(125) + \cos(204) + \cos(300) = ?$

$\cos(180 \pm x) = -\cos x$

$\cos(125) = \cos(180 \mp 55)$

$= -\cos(55)$

$\cos(204) = \cos(180+24)$

$= -\cos(24)$

also $\cos(300) = \cos(360-60)$

$= \cos(60) = 1/2$

so $\cancel{\cos(24)} + \cancel{\cos(55)} - \cancel{\cos(55)} - \cancel{\cos(24)} + 1/2$

$= 1/2$ Ans

4

40) $\sin(2x) = 2\sin x . \cos x$

$\sin\left(2\left(\frac{x}{2}\right)\right) = 2\sin\frac{x}{2} . \cos\frac{x}{2}$

41) $\frac{1-\cos(2x)}{\sin(2x)}$

$= \frac{1-(1-2\sin^2 x)}{2\sin x \cos x}$

$= \frac{\cancel{1}-\cancel{1}+2\sin^2 x}{2\sin x \cos x}$

$= \frac{\cancel{2}\sin^{\cancel{2}} x}{\cancel{2}\cancel{\sin x}.\cos x} = \tan x$

42) Identity.

$\cos(2\theta) = 1 - 2\sin^2(\theta)$

$\Rightarrow 1 - \cos(2\theta) = 2\sin^2(\theta)$

put $\theta = 2x$

$1 - \cos(4x) = 2\sin^2(2x)$

43) $\cos(2x) = 2\cos^2 x - 1$

$\cos(2x) + 1 = 1 + (2\cos^2 x - 1)$

$= 2\cos^2 x$

44) $\sqrt{\frac{1-\cos(2x)}{1+\cos(2x)}}$

$= \sqrt{\frac{1-(1-2\sin^2 x)}{1+(2\cos^2 x - 1)}}$

$= \sqrt{\frac{\cancel{2}\sin^2 x}{\cancel{2}\cos^2 x}} = \tan x$

45) use : $\sin\alpha - \sin\beta$

$= 2\cos\left(\frac{\alpha+\beta}{2}\right) . \sin\left(\frac{\alpha-\beta}{2}\right)$

46) $\cos(4x) = \cos(2(2x))$

$= \cos^2(2x) - \sin^2(2x)$

47) $\sin 80^\circ + \sin 20^\circ$

$= 2\sin\left(\frac{80+20}{2}\right) . \cos\left(\frac{80-20}{2}\right)$

$= 2\sin(50^\circ) . \cos(30^\circ)$

48) $\cos^4 x - \sin^4 x$

$= (\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x)$

$= (1)(\cos 2x) = \cos(2x)$

49) $\tan(2x) = \frac{2\tan x}{1-\tan^2 x}$

50) $\sqrt{1+\sin x}$

write 1 as,

$\sin^2\frac{x}{2} + \cos^2\frac{x}{2} = 1$

and

$\sin x = 2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)$

so

$= \sqrt{\sin^2\frac{x}{2} + \cos^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2}}$

$= \sqrt{\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^{\cancel{2}}}$

$= \sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)$ Ans

51) $\sin(3x) = 3\sin x - 4\sin^3 x$

52) $\cos(3x)$

$= 4\cos^3 x - 3\cos x$

53) $(\sin\theta + \cos\theta)^2$

$= \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta$

$= 1 + 2\sin\theta\cos\theta$

$= 1 + \sin(2\theta)$

54) $\tan(270+\theta) = -\cot(\theta)$

55) $\dfrac{\cos(\beta)+\cos(3\beta)+\cos(5\beta)}{\sin(\beta)+\sin(3\beta)+\sin(5\beta)}$

$\Rightarrow \cos(\beta)+\cos(5\beta)$

$= 2\cos\left(\frac{\beta+5\beta}{2}\right)\cdot\cos\left(\frac{\beta-5\beta}{2}\right)$

$= 2\cos\left(\frac{6\beta}{2}\right)\cdot\cos\left(-\frac{4\beta}{2}\right)$

$= 2\cos(3\beta)\cdot\cos(2\beta)$

$\Rightarrow \sin(\beta)+\sin(5\beta)$

$= 2\sin\left(\frac{\beta+5\beta}{2}\right)\cos\left(\frac{\beta-5\beta}{2}\right)$

$= 2\sin(3\beta)\cos(2\beta)$

put values

$= \dfrac{2\cos(3\beta).\cos(2\beta)+\cos(3\beta)}{2\sin(3\beta).\cos(2\beta)+\sin(3\beta)}$

$= \dfrac{\cos(3\beta)(\cancel{2\cos 2\beta+1})}{\sin(3\beta)(\cancel{2\cos 2\beta+1})}$

$= \cot(3\beta)$ Ans

56) $\dfrac{\cos(10)+\cos(30)+\cos(50)}{\sin(10)+\sin(30)+\sin(50)}$

$= \dfrac{2\cos(30)\cos(20)+\cos(30)}{2\sin(30)\cos(20)+\cos(30)}$

$= \dfrac{\cos(30)(\cancel{2\cos(20)+1})}{\sin(30)(\cancel{2\cos(20)+1})}$

$= \cot(30) = \sqrt{3}$

57) $\cos(90)-\cos(30)$

use $\cos\alpha - \cos\beta$

$= -2\sin\left(\frac{\alpha+\beta}{2}\right).\sin\left(\frac{\alpha-\beta}{2}\right)$

58) same as 57

59) $-2\sin(120).\sin(70)$

use $2\sin\alpha.\sin\beta$

$= \cos(\alpha-\beta) - \cos(\alpha+\beta)$

so $-2\sin(120).\sin(70)$

$= -[\cos(120-70) - \cos(120+70)]$

$= -[\cos(50) - \cos(90)]$

$= \cos(90) - \cos(50)$

60) $\tan(3x) =$

$\dfrac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$

61) $\sin(13+87)$

$= \sin(100)$

$= \sin(90+10^\circ)$

$= \cos(10)$ none

62) $\cos(30+20)$

$= \cos(50)$ c-option

63) $\sin(180-\theta)$

$= \sin\theta$

64) $\cos(180-\theta)$

$= -\cos(\theta)$

65) $\cos(120) = \cos(180-60)$

$= -\cos(60) = -1/2$

66) $\dfrac{\cos(75)+\cos(15)}{\sin(75)+\sin(15)}$

$= \dfrac{2\cos\left(\frac{75+15}{2}\right).\cos\left(\frac{75-15}{2}\right)}{2\sin\left(\frac{75+15}{2}\right).\cos\left(\frac{75-15}{2}\right)}$

$= \dfrac{\cancel{2}\cos(45).\cancel{\cos(30)}}{\cancel{2}\sin(45).\cancel{\cos(30)}}$

$= \dfrac{\cos(45)}{\sin(45)} = \dfrac{1/\cancel{\sqrt{2}}}{1/\cancel{\sqrt{2}}} = 1$

$\dfrac{\cos(75)+\cos(15)}{\sin(75)-\sin 15}$

$= \dfrac{2\cancel{\cos(45)}.\cos(30)}{2\cancel{\cos(45)}.\sin(30)}$

$= \dfrac{\cos(30)}{\sin(30)} = \dfrac{\sqrt{3}/2}{1/2} = \sqrt{3}$

67) $\cot(\alpha-\beta) =$

$\dfrac{(\cot\alpha\cot\beta+1)}{(\cot\beta-\cot\alpha)}$ Ans

~~68) cos(360~~

68)

$\cos(36) = \frac{1+\sqrt{5}}{4}$

69) $2\cos^2 15 - 1 = ?$

$2\cos^2\theta - 1 = \cos 2\theta$

$2\cos^2 15 - 1 = \cos(2\times 15)$

$= \cos(30) = \frac{\sqrt{3}}{2}$

70) $\frac{\sin(36)+\sin(54)}{\cos(36)+\cos(54)}$

$= \frac{2\sin\left(\frac{36+54}{2}\right).\cos\left(\frac{36-54}{2}\right)}{2\cos\left(\frac{36+54}{2}\right).\cos\left(\frac{36-54}{2}\right)}$

$= \frac{2\sin(45)\cos(-9)}{2\cos(45).\cos(-9)}$

$= \frac{\cancel{2}\sin(45)\cancel{\cos(9)}}{\cancel{2}\cos(45)\cancel{\cos(9)}}$

$= \tan(45) = 1$

71) $\frac{\cos^3(x) - \sin^3(x)}{\cos(x) - \sin(x)}$

$= \frac{\cancel{(\cos x - \sin x)}(\cos^2 x + \cos x \sin x + \sin^2 x)}{\cancel{(\cos x - \sin x)}}$

$= (\cos^2 x + \sin^2 x + \cos x \sin x)$

$= 1 + \cos x \sin x$

72) $\sin(70+20)$

$= \sin(90) = 1$

73) $\sin\left(\frac{\overset{2}{\cancel{4}}\pi}{\cancel{2}} + \theta\right) = \sin(2\pi + \theta)$

$= \sin(\theta)$

74) $\sin(450+30) = \sin(480)$

$= \sin(360+120)$

$= \sin(120)$

$= \sin(180-60) = \sin 60$

$= \sqrt{3}/2$

75) $\sin^2(3x) + \cos^2(3x) = 1$

76) $\frac{\tan^2 35}{\cot^2 55} - \frac{\sin^2 17}{\cos^2 73}$

$\tan(35) = \cot(90-35)$

$= \cot(55)$

$\sin(17) = \cos(90-17)$

$= \cos(73)$

$= \frac{\cancel{\cot^2(55)}}{\cancel{\cot^2(55)}} - \frac{\cancel{\cos^2(73)}}{\cancel{\cos^2(73)}}$

$= 1 - 1 = 0$

77) $\sqrt{1-\sin(x)}$

$1 = \sin^2\frac{x}{2} + \cos^2\frac{x}{2}$

$\sin x = 2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)$

$\sqrt{1-\sin x}$

$= \sqrt{\sin^2\frac{x}{2} + \cos^2\frac{x}{2} - 2\sin\frac{x}{2}\cos\frac{x}{2}}$

$= \sqrt{\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)^{\cancel{2}}} =$

$= \left|\cos\frac{x}{2} - \sin\frac{x}{2}\right|$

or $= \left(\sin\frac{x}{2} - \cos\frac{x}{2}\right)$

78) holds true for all real values of x so Infinite

79) $\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$

$180° \le x \le 360°$

$90° \le \frac{x}{2} \le 360°$

so

$\sin\frac{x}{2} = +\sqrt{\frac{1-\cos x}{2}}$

80) $\sin\theta = 1/2$

$\tan\theta = -1/\sqrt{3}$

since $\sin\theta$ is +ve and $\tan\theta$ is −ve

θ lies in II Q

$\sin\theta = \frac{1}{2} \Rightarrow \theta = \pi/6$

$\tan\theta = 1/\sqrt{3} \Rightarrow \theta = \pi/6$

since θ in 2nd Q

so, $\pi - \frac{\pi}{6} = \frac{5\pi}{6}$

81) $\frac{\cos(x)}{1-\sin x} - \frac{\cos(x)}{1+\sin x} =$

$\frac{(1+\sin x)\cdot\cos x - (1-\sin x)\cdot\cos x}{(1-\sin x)(1+\sin x)}$

$= \frac{\cos x + \sin x\cos x - (\cos x - \cos x \sin x)}{1-\sin^2 x}$

$= \frac{\cancel{\cos x} + \sin x\cos x - \cancel{\cos x} + \cos x \sin x}{\cos^2 x}$

$= \frac{2\sin x\cdot\cancel{\cos x}}{\cos^{\cancel{2}} x} = 2\tan x$

82) $\frac{\sin(90-x)}{\cos(90+x)} - \frac{\operatorname{cosec}(90-x)}{\sec(90+x)}$

$= \frac{\cos(x)}{-\sin(x)} + \frac{\sec(x)}{+\csc(x)}$

$= -\cot x + \frac{\sin x}{\cos x}$

$= -\cot x + \tan x$

83) $1\text{rad} = 57°17'45''$

or $1\text{rad} = \frac{180°}{\pi}$ so

A, B both

84) $35.39° = 35° + 0.39°$

$= 35° + 0.39 \times 60$

$= 35° + (23.4)'$

$= 35° + 23' + 0.4'$

$= 35° + 23' + (0.4\times 60)''$

$= 35° + 23' + 24''$

so $35.39° = 35°23'24''$

85) $\frac{\text{rad}}{\text{min}} = \frac{2\pi}{60}$

$\frac{x}{35} = \frac{2\pi}{60}$

$x = \frac{2\pi}{60} \times 35$

$= 7\pi/6$

86) hour hand moves

$\frac{360°}{12\text{hr}} = 30 \text{ degree/hr}$

so $135/30 = 4.5\text{hr}$

or $4\frac{1}{2}$ hour

87) $\theta = 8.329 \text{ rad}$

$= 477.28$ degree

$477.28 - 360 = 117.28$

lies in 2^{nd} Q

88) $\frac{123\pi}{2} = \frac{122\pi + \pi}{2}$

$= \frac{\overset{61}{\cancel{122}}\pi}{\cancel{2}} + \frac{\pi}{2}$

$= 60\pi + \pi + \frac{\pi}{2}$

$= 2\pi(30) + \pi + \frac{\pi}{2} = 3\pi/2$

89) 4rth Q

$270° < \theta \leq 360°$

$\frac{270°}{3} < \frac{\theta}{3} < \frac{360°}{3}$

$90° < \frac{\theta}{3} < 120°$

2nd Q Ans

90) θ lies in 3^{rd} Q

$180 < \theta < 270$

$180 \times \frac{2}{3} < \frac{2\theta}{3} < 270 \times \frac{2}{3}$

$120° < \frac{2\theta}{3} < 180°$

2nd Quadrant

91) All of these

92) $2\pi - \theta$

93) $\cos\theta$ is negative in II and III Q

$\csc\theta$ is positive in I and II Q

so terminal side lies in II Q

94) $\sin 75° = \frac{\sqrt{3}+1}{2\sqrt{2}}$

95) $\tan 15^\circ = \dfrac{\sqrt{3}-1}{\sqrt{3}+1}$

96) $\dfrac{\tan 13 + \tan 32}{1 - \tan 13 \cdot \tan 32}$

$= \tan(13+32) = \tan 45$

$= 1$

97) which is $\tan\theta$

a) $\dfrac{\sin(90-\theta)}{\cos(90-\theta)} = \dfrac{\cos\theta}{\sin\theta} = \cot\theta$

b) $\dfrac{1}{\cot\theta} = \tan\theta$ ✓

c) $\tan(180+\theta) = \tan\theta$ ✓

both B and C

98) $\sqrt{\dfrac{1-\sin\theta}{1+\sin\theta} \cdot \dfrac{1-\sin\theta}{1-\sin\theta}}$

$= \sqrt{\dfrac{(1-\sin\theta)^2}{1-\sin^2\theta}}$

$= \dfrac{\sqrt{(1-\sin\theta)^2}}{\sqrt{\cos^2\theta}} = \dfrac{1-\sin\theta}{\cos\theta}$

$= \dfrac{1}{\cos\theta} - \dfrac{\sin\theta}{\cos\theta} = \sec\theta - \tan\theta$

99) $\tan(\pi-\theta) \cdot \cos\left(\frac{\pi}{2}-\theta\right) + \cos(\pi-\theta) = -\tan\theta \cdot \sin\theta$

$+ (-\cos\theta) = -\dfrac{\sin\theta}{\cos\theta} \cdot \sin\theta$

$- \cos\theta$

$= \dfrac{-(\sin^2\theta + \cos^2\theta)}{\cos\theta}$

$= \dfrac{-1}{\cos\theta} = -\sec\theta$

100) • $\sin^2(2x) = \dfrac{1-\cos(4x)}{2}$ ✓

• $\cos^2(2x) \cdot \tan^2(2x) = \cancel{\cos^2(2x)} \cdot \dfrac{\sin^2(2x)}{\cancel{\cos^2(2x)}} = \sin^2(2x)$ ✓

• $4\sin^2(x)\cos^2 x$

$= (2\sin x\cos x)^2 = \sin^2(2x)$ ✓

all of these

101) $\sin\alpha = \dfrac{1}{\sqrt{5}}$, $\sin\beta = \dfrac{1}{\sqrt{10}}$

$\cos\alpha = \sqrt{1-\left(\frac{1}{\sqrt{5}}\right)^2} = \dfrac{2}{\sqrt{5}}$

$\cos\beta = \sqrt{1-\left(\frac{1}{\sqrt{10}}\right)^2} = \dfrac{3}{\sqrt{10}}$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= \dfrac{1}{\sqrt{5}} \cdot \dfrac{3}{\sqrt{10}} + \dfrac{2}{\sqrt{5}} \cdot \dfrac{1}{\sqrt{10}}$

$= \dfrac{5}{\sqrt{50}} = \dfrac{\cancel{5}}{5\sqrt{2}} = \dfrac{1}{\sqrt{2}}$

$(\alpha+\beta) = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45^\circ = \dfrac{\pi}{4}$

102) $\sin x = \frac{1}{2} \Rightarrow x = 30^\circ$

$2x = 60^\circ$, $\tan 60 = \sqrt{3}$

103) $\sin\theta = \dfrac{4 \to \text{Per}}{5 \to \text{Hyp}}$

Base $= \sqrt{\text{Hyp}^2 - \text{Per}^2} = \sqrt{5^2 - 4^2}$

$= \sqrt{9} = 3$

$\cos\theta = \dfrac{\text{Base}}{\text{hyp}} = \dfrac{-3}{5}$ $\theta \in$ IIQ

$\cos\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1+\cos\theta}{2}} = \pm\sqrt{\dfrac{1+\left(-\frac{3}{5}\right)}{2}}$

$= +\sqrt{\dfrac{\frac{2}{5}}{2}} = +\sqrt{\dfrac{1}{5}} = \dfrac{1}{\sqrt{5}}$

$= \dfrac{1}{\sqrt{5}} = \dfrac{\sqrt{5}}{5}$

104) $\sin\theta = 4/5$

$\sin(3\theta) = 3\sin\theta - 4\sin^3\theta$

$= 3 \cdot \dfrac{4}{5} - 4\left(\dfrac{4}{5}\right)^3$

$= \dfrac{12}{5} - \dfrac{256}{125} = \dfrac{44}{125}$

105) $\cos\theta = 4/5$

$\tan\left(\dfrac{\theta}{2}\right) = -\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}}$

$= -\sqrt{\dfrac{1-\frac{4}{5}}{1+\frac{4}{5}}} = -\sqrt{\dfrac{1/5}{9/5}} = \dfrac{-1}{3}$

106) $r\sin(\theta+\alpha) =$

$r(\sin\theta\cos\alpha + \cos\theta\sin\alpha)$

$= r\cos\alpha\sin\theta + r\sin\alpha\cos\theta$

compare coefficients of $\sin\theta$ and $\cos\theta$

$r\cos\alpha = \sqrt{3}$, $r\sin\alpha = 1$

$r^2\cos^2\alpha + r^2\sin^2\alpha = 1^2 + (\sqrt{3})^2$

$r^2(\sin^2\alpha+\cos^2\alpha) = 1+3$

$r^2(1) = 4 \Rightarrow r = 2$

$\frac{r\sin\alpha}{r\cos\alpha} = \frac{1}{\sqrt{3}}$ $\quad \tan\alpha = \frac{1}{\sqrt{3}}$

$\alpha = 30^\circ$

$r\sin(\theta+\alpha) = 2\sin(\theta+30^\circ)$

107) same as 106

or you can backsolve

108) $\alpha+\beta = \gamma$

$\tan(\alpha+\beta) = \tan\gamma$

$\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta} = \tan\gamma$

$\tan\alpha+\tan\beta = \tan\gamma(1-\tan\alpha\tan\beta)$

$\tan\alpha+\tan\beta = \tan\gamma - \tan\alpha\tan\beta\tan\gamma$

$\tan\alpha\tan\beta\tan\gamma = \tan\gamma - \tan\alpha - \tan\beta$ Ans

109) $\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

$= \frac{\frac{m}{m+1} + \frac{1}{2m+1}}{1 - \frac{m}{m+1}\cdot\frac{1}{2m+1}}$

$= \frac{\frac{2m^2+m+m+1}{\cancel{(m+1)(2m+1)}}}{\frac{(m+1)(2m+1)-m}{\cancel{(m+1)(2m+1)}}}$

$= \frac{2m^2+2m+1}{2m^2+m+2m+1-m} = \frac{2m^2+2m+1}{2m^2+2m+1}$

$\tan(\alpha+\beta) = 1$

$\Rightarrow \alpha+\beta = 45^\circ$ or $\pi/4$

110) $\frac{\sin(75)+\sin(15)}{\cos(75)+\cos(15)}$

$= \frac{\cancel{2}\sin\left(\frac{75+15}{2}\right)\cancel{\cos\left(\frac{75-15}{2}\right)}}{\cancel{2}\cos\left(\frac{75+15}{2}\right)\cancel{\cos\left(\frac{75-15}{2}\right)}}$

$= \frac{\sin(45)}{\cos(45)} = \tan 45 = 1$

111) $\left(\sin\frac{\alpha}{2} - \cos\frac{\alpha}{2}\right)^2$

$= \sin^2\frac{\alpha}{2} + \cos^2\frac{\alpha}{2} - 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}$

$= 1 - \sin\left(\cancel{2}\cdot\frac{\alpha}{\cancel{2}}\right)$

$= 1 - \sin\alpha$

112) $\frac{1+\cos x+\cos 2x}{\sin x+\sin 2x}$

$= \frac{\cancel{1}+\cos x + 2\cos^2 x - \cancel{1}}{\sin x + 2\sin x\cos x}$

$= \frac{\cos x\,\cancel{(1+2\cos x)}}{\sin x\,\cancel{(1+2\cos x)}}$

$= \cot x$

113) $\theta = 43^\circ + \frac{42}{60}$

$= 43^\circ + 0.7^\circ = 43.7^\circ$

convert in rad

$43.7^\circ = \frac{\pi}{180} \times 43.7$ rad

$s = r\theta$

$s = 2 \times \frac{\pi 43.7}{180}$ ~~rad~~

$= \frac{43.7}{90}\pi$

114) $\sin 44^\circ \cos 25^\circ$

use $\sin(A).\cos(B)$

$= \frac{1}{2}\left[\sin(A+B)+\sin(A-B)\right]$

115) $\sin x + 2\sin 3x + \sin 5x$

$\sin x + \sin 5x$

$= 2\sin\left(\frac{x+5x}{2}\right)\cos\left(\frac{x-5x}{2}\right)$

$= 2\sin 3x\cos(-2x)$

put back

$= 2\sin(3x)\cos(2x) + 2\sin(3x)$

$= 2\sin(3x)(\cos(2x)+1)$

use $\cos 2x = 2\cos^2 x - 1$

$\cos 2x + 1 = 2\cos^2 x$

$= 2\sin(3x)(2\cos^2 x)$

$= 4\sin(3x)\cos^2(x)$

116) $\frac{\cos\theta - \cos(3\theta)}{\sin\theta + \sin(3\theta)}$

$= \frac{-\cancel{2}\cancel{\sin(2\theta)}\sin(-\theta)}{\cancel{2}\cancel{\sin(2\theta)}\cos(-\theta)}$

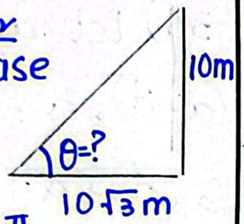
$= \frac{\sin\theta}{\cos\theta} = \tan(\theta)$

used sum to product

117) $\dfrac{\cos(\alpha+\beta)+\cos(\alpha-\beta)}{\sin(\alpha+\beta)+\sin(\alpha-\beta)}$

$= \dfrac{\cancel{2}\cos(\alpha)\cancel{\cos(\beta)}}{\cancel{2}\sin(\alpha)\cancel{\cos(\beta)}} = \cot(\alpha)$

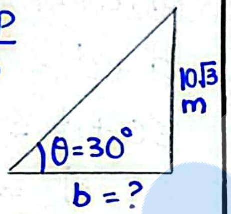
118) $\tan\theta = \dfrac{Per}{Base}$



$= \dfrac{\cancel{10}}{\cancel{10}\sqrt{3}} = \dfrac{1}{\sqrt{3}}$

$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$

119) $\tan(30^\circ) = \dfrac{P}{b}$



$\dfrac{1}{\sqrt{3}} = \dfrac{10\sqrt{3}}{b}$

$b = 10\sqrt{3}.\sqrt{3} = 30m$

120) $\dfrac{1-\cos(8\theta)}{2\sin^2 4\theta}$

$\because \cos(2x) = 1-2\sin^2 x$

$= \dfrac{2\cancel{\sin^2 4\theta}}{2\cancel{\sin^2 4\theta}} = 1$

121) $\dfrac{1+\cos(2x)}{\sin(2x)}$

$= \dfrac{\cancel{1}+(2\cos^2 x\cancel{-1})}{2\sin x\cos x} = \dfrac{\cancel{2}\cos^{\cancel{2}} x}{\cancel{2}\sin x\cancel{\cos x}}$

$= \dfrac{\cos(x)}{\sin(x)} = \cot(x)$

=> Application Of Trigonometry solution

1) Oblique Triangle

2) both A and B

3) Always acute

4) scalene Triangle

5) $d_1 = \sqrt{(3-0)^2+(0-0)^2}$

$= 3$

$d_2 = \sqrt{(3-3)^2+(4-0)^2}$

$= 4$

$d_3 = \sqrt{(3-0)^2+(4-0)^2}$

$= 5$ (scalene)

6) All

7) 6 parameter

8) $\sin\gamma = \dfrac{Per}{hyp} = \dfrac{AB}{AC}$

9) $\tan(60^\circ) = \dfrac{DA}{AB} = \dfrac{4}{AB}$

$\Rightarrow AB = \dfrac{4}{\sqrt{3}}$

$\tan(45) = \dfrac{4}{AC} \Rightarrow AC = 4$

$BC = AC - AB = 4 - \dfrac{4}{\sqrt{3}}$

10) $c^2 = a^2+b^2-2ab\cos\gamma$

11) $\dfrac{\sin(30)}{a} = \dfrac{\sin(60)}{x}$

$\dfrac{\sin(30)}{4} = \dfrac{\sin(60)}{x}$

$x = \dfrac{\sin(60)\times 4}{\sin(30)}$

$x = \dfrac{\sqrt{3}/2}{1/2}\times 4 = 4\sqrt{3}$ (none)

13) $\angle C = 180 - \angle A - \angle B$

$\angle C = 180-60-30 = 90^\circ$

$\sin(30) = \dfrac{AC}{10} \Rightarrow AC = 5$

$\sin(60) = \dfrac{BC}{10} \Rightarrow BC = 5\sqrt{3}$

$\Delta = \frac{1}{2}AC.BC = \frac{1}{2}.5.5\sqrt{3}$

$= \dfrac{25\sqrt{3}}{2}$

12) $\Delta = \frac{1}{2}ab\sin\gamma$

$\Delta = \frac{1}{2}8.10\sin(60)$

$\Delta = 40.\frac{\sqrt{3}}{2} = 20\sqrt{3}$

14) $\Delta = \dfrac{\sqrt{3}}{4}a^2 = \dfrac{\sqrt{3}}{\cancel{4}}\cancel{4^2}$

$\Delta = 4\sqrt{3}$

15) $\Delta = \dfrac{P^2}{\sqrt{3}} = \dfrac{36}{\sqrt{3}} = 12\sqrt{3}$

16) $\sqrt{r_1.r_2.r_3.r} = ?$

$= \sqrt{\Delta^{\cancel{2}}} = \Delta = rs$

17) Area $= rs$

$20 = 5s \Rightarrow s = 4$

Perimeter $= 2s = 2(4) = 8$

18) $s(s-a)(s-b)(s-c) = \Delta^2$

19) $r : R : r_1 = 1 : 2 : 3$

20) $R = \dfrac{b}{2\sin(\beta)} = \dfrac{\cancel{2}}{\cancel{2}\sin(30)} = 2$

$A = \pi R^2 = \pi(2)^2 = 4\pi$

21) $\cos\frac{\gamma}{2} = \sqrt{\frac{s(s-c)}{ab}}$

22) $\tan\frac{\alpha}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$

23) $3^2+4^2=5^2$
Right angled triangle
so $\Delta = \frac{1}{2}.3.4=6$

24) hyp of right angled triangle = diameter
$= 2r = 2\times 20$
$= 40m$

25) $R = \frac{a}{2\sin\alpha} = \frac{b}{2\sin\beta} = \frac{c}{2\sin\gamma}$
So all

26) $s = \frac{3+4+5}{2} = 6$
$A = \frac{1}{2}\,3.4 = 6$
$sr = A \Rightarrow r = \frac{A}{s} = \frac{6}{6} = 1$
check for options by putting values

27) $a-b+c = ?$
$a+b+c-b-b$ $(+,-b)$
$\times$ and $\div$ by 2
$= \left[\frac{a+b+c-2b}{2}\right]2$
$= \left(\frac{a+b+c}{2} - \frac{\cancel{2}b}{\cancel{2}}\right)2$
$= (s-b)2 = 2(s-b)$

28) use sum of 2 sides in Δ must be greater than the 3rd one.

29) let $x = 2$
so $2x+1 = 5$, $3x+4=10$
options
$a \Rightarrow 4$, $b \Rightarrow 16$, $c \Rightarrow 13$
$4+5>10$ false
$5+10>16$ false
c-option satisfies the condition

30) $10+12>x \Rightarrow x<22$
$10+x>12 \Rightarrow x>2$
$12+x>10 \Rightarrow x>-2$
since x represent a side length it must be positive.
combining,
$2<x<22$ $(3,4\ldots 21)$
Number of Integer values
$= 21-3+1 = 19$

31) Let angles are $x, 3x, 5x$
so $x+3x+5x=180$
$x = 20$
so $x=20$, $3x=60$, $5x=100$

32) $1:1:\underline{1}$

33) $m\angle\alpha = 90^\circ$

34) $m\angle\gamma = 180-\angle\alpha-\angle\beta$
$= 180-35-61 = 84$
so, scalene

35) $\frac{A_{squre}}{A_{triangle}} = \frac{\cancel{a^2}}{\frac{\sqrt{3}}{4}\cancel{a^2}}$
$= \frac{4}{\sqrt{3}}$ or $4:\sqrt{3}$

36) Let angles are $(a-d), a, (a+d)$
$(a-d)+a+(a+d) = 180$
$3a = 180 \Rightarrow a=60$
condition
$a+d = 2(a-d)$
$60+d = 2(60-d)$
$d = 20$
angles are
$a-d = 60-20 = 40$
$a = 60$
$a+d = 60+20 = 80.$

~~37)~~ 46) All of these

47) $r = \frac{\Delta}{s}$

48) All of these

49) S/r

50) $r:R = 1:2$

51) when $\alpha = 90^\circ$ then $r = s-a$
$r = 6-5 = \underline{1}$

52) A.P

=> Inverse Trigonometry Solution

1) Period of $f(kx)$

$= \dfrac{P \text{ of } f(x)}{|k|}$

2) $\mathbb{R}$

3) $\mathbb{R}$

4) $\mathbb{R} - \{(2n+1)\frac{\pi}{2}\}$ $n \in Z$

5) $\mathbb{R} - \{n\pi\}$ $n \in Z$

6) $\mathbb{R} - \{n\pi\}$ $n \in Z$

7) $\mathbb{R} - \{(2n+1)\frac{\pi}{2}\}$ $n \in Z$

8) $[-1, 1]$

9) $[-1.1]$

10) $\mathbb{R}$

11) $\mathbb{R}$

12) $-1 \geq y \geq 1$

or $\mathbb{R} - (-1, 1)$

13) 2π.

14) for $\tan x$ $\quad x \neq (2n+1)\frac{\pi}{2}$

for $\tan(2x)$ $\quad 2x \neq (2n+1)\frac{\pi}{2}$

$\Rightarrow x \neq (2n+1)\frac{\pi}{4}$

So Domain is

$\mathbb{R} - \{(2n+1)\frac{\pi}{4}\}$

15) $\mathbb{R} = \{(2n+1)\frac{3\pi}{2}\}$

16) $\mathbb{R}$ (Domain of $\sin x$ and $\cos x$ is always $\mathbb{R}$)

~~17)~~

17) $\mathbb{R} - \{\frac{n\pi}{2}\}$

18) Range of $cf(x)$

$= c(\text{Range of } f(x))$

Range of $y = 2\sec 4x$

$= 2[\text{Range of } \sec 4x]$

$= 2[-1 \geq y \geq 1]$

$= -2 \geq y \geq 2$

19) Period of $f(cx) = \dfrac{P \text{ of } f(x)}{|c|}$

so period of $30\sec\frac{x}{3}$

$= \dfrac{2\pi}{1/3} = 6\pi$

20) P of $\frac{-3}{4}\cos(5x)$

$= \dfrac{2\pi}{|5|} = \dfrac{2\pi}{5}$

21) period of

$y = A\sin(Bx+C)+D$

$= \dfrac{2\pi}{|B|}$ only so

period of $\sin\left(\frac{x}{4} + \frac{\pi}{2}\right)$

$= \dfrac{2\pi}{1/4} = 8\pi$

22) period of $f(x) = A\cos(Bx+C)$

$= \dfrac{2\pi}{|B|}$ so period is

$= \dfrac{2\pi}{1/4} = 8\pi$

23) $\sin^2 x = \dfrac{1-\cos(2x)}{2}$

$= \frac{1}{2} - \frac{1}{2}\cos(2x)$

perid of $\cos(ax)$

$= \dfrac{2\pi}{|a|}$ period is

$\dfrac{2\pi}{|2|} = \pi$

24) P of $\sin(3x)+\cos(4x)$

P of $\sin(3x) = \frac{2\pi}{3}$

P of $\cos(4x) = \frac{2\pi}{4} = \frac{\pi}{2}$

P of $\sin(3x) + \cos(4x)$

= L.C.M of periods

$= \{\frac{2\pi}{3}, \frac{\pi}{2}\} = 2\pi$

25) p of $\sec(3x) = \frac{2\pi}{3}$

P of $\cot(5x) = \frac{\pi}{5}$

Find L.C.M of both

L.C.M of $\{\frac{a}{b}, \frac{c}{d}, \frac{e}{f}\}$

$= \dfrac{\text{L.C.M of } \{a, c, e\}}{\text{H.C.F of } \{b, d, f\}}$

so period is 2π

26) P of $\sin 3x = \frac{2\pi}{3}$

P of $\cos 4x = \frac{\pi}{2}$

P of $\sin(3x) + \cos(4x) = 2\pi$

$f = \dfrac{1}{P} = \dfrac{1}{2\pi}$

27) Max of $a+b\cos\theta$

$= a+|b|$

Max of $1+2\cos x$

$= 1+|2| = 3$

28) Max of $a+b\sin\theta$

$= a+|b|$

Max of $1-2\sin x$

$= 1+|-2| = 1+2 = 3$

Min of $a+b\sin\theta$

$= a-|b| = 1-|-2| = -1$

29) Max $= a+b|$

$= 5 + \frac{3}{2} = \frac{10+3}{3}$

$= \frac{13}{2}$

30) Max $= 3$

Min $= -3$

Amplitude $= \frac{1}{2}(\text{Max}-\text{Min})$

$= \frac{1}{2}(3-(-3)) = \frac{6}{2} = 3$

31) Max of $a\sin\theta \pm b\cos\theta$

$= +\sqrt{a^2+b^2}$ Min $= -\sqrt{a^2+b^2}$

Max $= \sqrt{9+16} = 5$

Min $= -\sqrt{9+16} = -5$

$A = \frac{1}{2}(5+5) = 5$

32) Max $= 3$ of $1+2\sin x$

Min $= -1$

$M' = \frac{1}{M} = \frac{1}{3}$

33) Min $= -\sqrt{a^2+b^2}$

$= -\sqrt{6^2+8^2} = -10$

34) Max $= \sqrt{a^2+b^2}$

$= \sqrt{1^2+1^2} = \sqrt{2}$

35) Max and Min of $3+2\cos x$ are $5, +1$

$M' = \frac{12}{M} = \frac{12}{+1} = +12$

36) $M = 10$, $m = 4$

$M, m > 0$ so $M' = \frac{14}{m} = \frac{14}{4}$

and $m' = \frac{14}{M} = \frac{14}{10} = \frac{7}{5}$

37) $M = \sqrt{3^2+4^2} = 5$

$m = -5$

$M' = \frac{1}{M} = \frac{1}{5}$

38) $30°$ or $\pi/6$

39) $\sin^{-1}\left(\frac{1}{2}\right) + \cos^{-1}\left(\frac{\sqrt{4}}{2}\right)$

$= 30° + \cos^{-1}(1)$

$= 30° + 0° = 30°$

40) Neither even nor odd

41) $\cos^{-1}(-x) = \pi - \cos^{-1}(x)$

$\cos^{-1}\left(-\frac{1}{2}\right) = \pi - \cos^{-1}\left(\frac{1}{2}\right)$

$= \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

42) $[-1,1]$

43) $\sin^{-1}(-x) = -\sin^{-1}(x)$

$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$= -60°$

44) both are odd

45) $\operatorname{cosec}^{-1}(-2) = -\csc^{-1}(2)$

$= -\sin^{-1}\left(\frac{1}{2}\right)$

$= -30°$

46) $\sec^{-1}(-x) = \pi - \sec^{-1}(x)$

$\sec^{-1}\left(\frac{-2}{\sqrt{3}}\right) = \pi - \sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$

$= \pi - \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$= \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

47) $\csc^{-1}(x) + \sec^{-1}(x) = \frac{\pi}{2}$

48) $\sin^{-1}(x) + \cos^{-1}(x) = \frac{\pi}{2}$

so $\sin^{-1}(0.03) + \cos^{-1}(0.03) = \frac{\pi}{2}$

49) $\cot^{-1}(x) = \tan^{-1}\left(\frac{1}{x}\right)$

$\tan^{-1}(-x) + \cot^{-1}\left(\frac{-1}{x}\right) =$

$\tan^{-1}(-x) + \tan^{-1}(-x)$

$= 2\tan^{-1}(-x)$

50) $\sin\left(\frac{\pi}{2} - x\right) = \cos x$

$\sin\left(\frac{\pi}{2} - \cos^{-1}(x)\right)$

$= \cancel{\cos}(\cos^{-1}(x))$

$= x$

51) $\sin^{-1}(\sin\pi)$

$= \sin^{-1}(0)$ 1st reference angle is $0°$

2nd is $\pi - 1^{st} = \pi - 0 = \pi$

also add the period

$0° + 2\pi = 2\pi$

so all

52) principle sin so $0°$ is the only sol

53) $\tan^{-1}\left(\frac{1}{\cot x}\right)$

$= \cancel{\tan^{-1}}(\cancel{\tan}(x)) = x$

54) $2\tan^{-1}(x) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

55) $\tan^{-1}(x) + \tan^{-1}\left(\frac{1}{x}\right)$

$= \tan^{-1}(x) + \cot^{-1}(x)$

$= \pi/2$

56) cosine of $\cos^{-1}(0.5)$

$= \cancel{\cos}(\cancel{\cos^{-1}}(0.5)) = 1/2$

57) $\sin^{-1}(-x) = -\sin^{-1}(x)$

$\Rightarrow \sin^{-1}(-1) = -\sin^{-1}(1)$

$= -90°$

58) $\cos^{-1}(1) = 0°$

59) $\cos^{-1}\left(\frac{-1}{2}\right) = \pi - \cos^{-1}\left(\frac{1}{2}\right)$

$= \pi - \frac{\pi}{3} = 2\pi/3$

60) $\sec^{-1}(-\sqrt{2})$

$= \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

$= \pi - \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$= \pi - \frac{\pi}{4} = \frac{3\pi}{4} = 135°$

61) $\sin(360-\theta) = -\sin\theta$

$\sin(360-60) = -\sin 60$

$\sin^{-1}(\sin(300))$

$= \sin^{-1}(\sin(-60))$

$= -60°$

62) $y = \sin\left(\frac{x}{2}\right)$ has period 4π

No of Periods = $\frac{\text{Intervals}}{\text{period}}$

$= \frac{2\cancel{\pi}}{4\cancel{\pi}} = 1/2$

= half

63) $y = \cos(3x)$ has period $\frac{2\pi}{3}$

No of periods = $\frac{\cancel{2\pi}}{\frac{\cancel{2\pi}}{3}}$

$= 3$

64) $y = A\cos(Bx+C)$

$A \Rightarrow$ Amplitude

65) $\cos x = -\frac{\sqrt{3}}{2}$

$x = \pi - \frac{\pi}{6} = \frac{5\pi}{6} \rightarrow$ 2nd Q

$x = \pi + \frac{\pi}{6} = \frac{7\pi}{6} \rightarrow$ 3rd Q

$\left\{\frac{5\pi}{6} + 2n\pi\right\} \cup \left\{\frac{7\pi}{6} + 2n\pi\right\}$

66) $\tan\left(\cos^{-1}\left(-\frac{1}{2}\right)\right)$

$= \tan\left(\pi - \cos^{-1}\left(\frac{1}{2}\right)\right)$

$= \tan\left(\pi - \frac{\pi}{3}\right) = -\tan\left(\frac{\pi}{3}\right)$

$= -\sqrt{3}$ Ans

67) $\tan\theta = \frac{\sqrt{3}}{3}$

$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$

sol $\Rightarrow \left\{\frac{\pi}{6} + n\pi\right\}$

68) $\pi\sin x = 2x$ is true for $x = 0$

$\pi\sin(0) = 2(0)$

$0 = 0$ ✓

69) $\cos^{-1}\left(-\frac{1}{2}\right)$ $\pi - \frac{\pi}{3}$

principle value is $\frac{2\pi}{3}$

for 3rd Q. $\pi + \frac{\pi}{3} = \frac{4\pi}{3}$

$\left\{\frac{2\pi}{3} + 2n\pi\right\} \cup \left\{\frac{4\pi}{3} + 2n\pi\right\}$

70) $\cos(\sin^{-1}(x))$

$P = x$, $hyp = 1$

$B = \sqrt{1^2 - x^2} = \sqrt{1-x^2}$

$\cos\theta = \frac{\sqrt{1-x^2}}{1}$

$= \cancel{\cos}(\cancel{\cos^{-1}}\sqrt{1-x^2})$

$= \sqrt{1-x^2}$

71) $\sin(\cos^{-1}(x))$

$\frac{B}{H} = \frac{x}{1}$ Per $= \sqrt{1-x^2}$

$= \cancel{\sin}\left(\cancel{\sin^{-1}}\frac{\sqrt{1-x^2}}{1}\right)$

$= \sqrt{1-x^2}$

72) $\sin^{-1}(x) = \frac{\pi}{4}$

$x = \frac{\sqrt{2}}{2}$

$\cos^{-1}(x) = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$

$= \pi/4$

73) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) + 3\sin^{-1}\left(\frac{1}{2}\right)$

$= \frac{\pi}{6} + 3\cdot\frac{\pi}{6} = \frac{\pi}{6} + \frac{\pi}{2}$

$= \frac{\pi + 3\pi}{6} = \frac{4\pi}{6} = \frac{2\pi}{3}$

74) $\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\left(\frac{x-y}{x+y}\right)$

$= \tan^{-1}\left[\frac{\frac{x}{y} - \frac{x-y}{x+y}}{1 + \frac{x}{y}\cdot\frac{x-y}{x+y}}\right]$

$= \tan^{-1}\left[\frac{\frac{x(x+y) - y(x-y)}{\cancel{y(x+y)}}}{\frac{y(x+y)+x(x-y)}{\cancel{y(x+y)}}}\right]$

$= \tan^{-1}\left[\frac{x^2 + \cancel{xy} - \cancel{xy} + y^2}{\cancel{xy} + y^2 + x^2 - \cancel{xy}}\right]$

$= \tan^{-1}\left(\frac{\cancel{x^2+y^2}}{\cancel{x^2+y^2}}\right) = \tan^{-1}(1)$

$= \pi/4$

75) $\cos(\tan^{-1}[\sin(\cot^{-1}(x))])$

$\cot x = \frac{x \to B}{1 \to A}$ $H = \sqrt{x^2+1}$

$\sin\theta = \frac{1}{\sqrt{x^2+1}}$

$= \cos\left(\tan^{-1}\frac{1}{\sqrt{x^2+1}}\right)$

$\tan x = \frac{1 \to P}{\sqrt{x^2+1} \to B}$ ~~$H = \sqrt{}$~~

$Hyp = \sqrt{1^2 + \sqrt{(x^2+1)}^2}$

$= \sqrt{x^2+2}$

$\cos x = \frac{\sqrt{x^2+1}}{\sqrt{x^2+2}}$

$\frac{\sqrt{x^2+1}}{\sqrt{x^2+2}}$ Ans

76) use $\tan^{-1}(A) + \tan^{-1}(B)$

$= \tan^{-1}\left(\frac{a+b}{1-a\cdot b}\right)$

77) $\sin^{-1}\left(\sin\frac{5\pi}{3}\right)$

$= -\pi/3$

78) use $\tan^{-1}(A) + \tan^{-1}(B)$

$= \tan^{-1}\left(\frac{A+B}{1-AB}\right)$

79) $2\sin^{-1}(x) - \cos^{-1}(x) = \frac{\pi}{2}$

$2\sin^{-1}(x) - \left(\frac{\pi}{2} - \sin^{-1}(x)\right) = \frac{\pi}{2}$

$2\sin^{-1}(x) - \frac{\pi}{2} + \sin^{-1}(x) = \frac{\pi}{2}$

$3\sin^{-1}(x) = \frac{\pi}{2} + \frac{\pi}{2}$

$3\sin^{-1}(x) = \pi$

$\sin^{-1}(x) = \frac{\pi}{3}$

$x = \frac{\sqrt{3}}{2}$

80) $\tan^{-1}(x) + \tan^{-1}\left(\frac{1}{x}\right)$

$= \tan^{-1}(x) + \cot^{-1}(x) = \pi/2$

$= 90^\circ$